# §3. Examples of SCFTs

In this paragraph, we will look at examples of SCFTs in 3, 4, 5 and 6 dimensions.

## §3.1 3d $\mathcal{N}=2$ ($\mathcal{N}=4$) SCFTs

We want to start with ordinary supersym. Lagrangians which flow to an IR SCFT fixed point.

$\mathcal{N}=2$ supersymmetry algebra in $d=3$:

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0,$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu + 2i\varepsilon_{\alpha\beta} Z$$

$Z$ is real central term $\rightarrow$ arises from momentum $P_3$ along $S^1$ in a dimensional reduction from 4d $\mathcal{N}=1$.

R-sym. group: $U(1)_R$

superfields:

- chiral superfield $X$ with $\bar{Q}_{\dot\alpha} X = Q_\alpha \bar{X} = 0$
- vector superfield $V = V^\dagger$
- linear multiplet $\Sigma$: $\varepsilon^{\alpha\beta} Q_\alpha Q_\beta \Sigma = \varepsilon^{\dot\alpha\dot\beta} \bar{Q}_{\dot\alpha} \bar{Q}_{\dot\beta} \Sigma = 0$
  (lowest component: real scalar)

All states satisfy BPS bound: $M \geq |Z|$

Wess-Zumino theories:

$$\mathcal{L} = \int d^4\theta\, K(X,\bar{X}) + \left(\int d^2\theta\, W(X) + h.c\right)$$

$X$ has scaling dimension $\frac{1}{2}$

$W = X^3$ flows to interacting SCFT fixed point

All operators satisfy: $\Delta \geq |R|$

$\rightarrow$ saturated for chiral/antichiral fields (see tables in §2.2)

For $W = X^3$, $R(X) = \frac{2}{3} \rightarrow \Delta(X) = \frac{2}{3}$

"exact anomalous dimension" of $X$ at IR fixed-point.

Gauge theories:

Vector multiplet $V$ of $d=3$ contains fields

- $A_m$ ($d=3$ vector potential)
- $\phi$ (real scalar in adjoint of $G$)
- $\lambda$ (complex fermion gaugino)

field strengths: $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$

$$\bar{W}_\alpha = -\frac{1}{4}DDe^{-V}\bar{D}_\alpha e^V$$

→ gauge Kinetic term:

$$\frac{1}{g^2}\int d^2\theta \, \mathrm{Tr}\, W_\alpha^2 + \text{h.c.}$$

$A_\mu$, $\phi$ are neutral under $U(1)_R$

$\lambda$ has $U(1)_R$ charge $+1$.

Fayet-Iliopoulos term $\underset{\substack{\uparrow \\ \text{real FI-parameter}}}{\xi}\int d^4\theta\, V$

moduli space of vacua:

1) "Coulomb branch" : $\langle\phi\rangle \neq 0$

$$G \longrightarrow U(1)^r, \quad r = \mathrm{rank}(G)$$

$\rightarrow \mathcal{M}_{\text{Coulomb}} = \mathbb{R}^r / \mathcal{W}$ ($\mathcal{W}$ is Weyl group)

$U(1)^r$ gauge fields can be dualized to scalars via $F^{(j)}_{\mu\nu} = \varepsilon_{\mu\nu\sigma}\partial^\sigma \gamma^j, \quad j = 1, \dots, r$

(this is done via a Lagrange multiplier

$$\mathcal{L} = -\frac{1}{g^2} F_{\mu\nu}F^{\mu\nu} + \gamma\,\partial_\mu F_{\nu\delta}\,\varepsilon^{\mu\nu\delta} + \dots$$

and $\dfrac{\delta \mathcal{L}}{\delta F_{\mu\nu}} = 0$ )

scalars $\gamma^j$ live on $r$-dimensional torus due to gauge instantons:

$$S_{top} = i\langle\gamma\rangle \underbrace{\int d^3x\, \varepsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho}}_{= k \in \mathbb{Z}}$$

$\rightarrow e^{iS_{top}}$ is periodic in $\gamma$ with period $2\pi$

The currents $J_\mu^{(j)} = \varepsilon_{\mu\nu\rho} (F^{\nu\rho})^{(j)}$ correspond to shifts of $\gamma^j \to$ generate "magnetic" $(U(1)_J)^r$ global symmetries.

$\Phi^j = \phi^j + i\gamma^j$ is chiral superfield

2) "Higgs branch":

can have matter multiplets in representations $R_f$ of gauge group

$$\to \sum_f \int d^4\theta \, Q_f^\dagger e^V Q_f$$

where $V$ includes a term $\phi \bar{\theta}\theta$

$$\to \sum_f |\phi Q_f|^2$$

$\langle\phi\rangle$ looks like a "real mass" for matter fields

Higgs branch: $\langle Q_f \rangle \neq 0$ (requires $\langle\phi\rangle = 0$)

$\to$ classical moduli space of vacua consists of distinct branches, Coulomb with $\langle\phi\rangle \neq 0$ and $\langle Q_f\rangle = 0$, and Higgs with $\langle Q_f\rangle \neq 0$ and $\langle\phi\rangle = 0$

charge assignments:

$U(1)_R$ can mix with other $U(1)$ global symmetries

$\rightarrow$ charge assigments at IR fixed points

<u>Linear multiplets:</u>

$\Sigma = \varepsilon^{\dot\alpha\beta} \overline{Q}_{\dot\alpha} Q_\beta V$, where $V$ is vector mult.

lowest component: scalar $\phi$ in $V$

We have $\varepsilon^{\alpha\beta} Q_\alpha Q_\beta \Sigma = \varepsilon^{\dot\alpha\dot\beta} \overline{Q}_{\dot\alpha} \overline{Q}_{\dot\beta} \Sigma = 0$

$\rightarrow$ is of type $A_2 \overline{A_2}$

$\rightarrow$ from §2.2 we know the deformation $Q\overline{Q}\mathcal{O}$ (flavor current) for $\mathcal{O}$ primary in $A_2\overline{A}_2 \left\{ \begin{matrix} (0) \\ \Delta_{\mathcal{O}} = 1 \end{matrix} \right\}$

concretely: $\Sigma = \phi + \bar\theta \sigma_\rho \theta \underbrace{F_{\mu\nu}\, \varepsilon^{\rho\mu\nu}}_{= \mathcal{J}^\rho}$

$\rightarrow$ gauge kinetic term $\frac{1}{g^2} \int d^4\theta\, \Sigma^2$ (D-term)

(irrelevant $\Delta > 3$, see §2.2)

Generally, for a conserved current $\mathcal{J}^\mu$, there is linear multiplet $\mathcal{J}$ including $\bar\theta \sigma^\mu \theta \mathcal{J}_\mu$

$\rightarrow$ $\mathcal{J}^\mu$ contributes to central charge $Z$.

<u>example:</u>

$$\mathcal{L} = \int d^4\theta\, X^\dagger e^{\tilde m \theta\bar\theta} X \quad , \; X \text{ chiral mult.}$$

$$= \tilde m |X|^2 + i \tilde m\, \varepsilon^{\alpha\beta}\, \overline{\psi}_\alpha \psi_\beta + \dots$$

$Z$ corresponds to $\mathcal{J}$ containing the global current under which $X$ is charged

$\rightarrow Z = \tilde{m} \rightarrow X$ is BPS

adding superpotential $W = mXY$ gives mass $M = \sqrt{\tilde{m}^2 + |m|^2} \rightarrow$ non-BPS !

<u>example 2:</u>

consider $U(1)_J$ symmetry corresponding to shifting the dual photon $\gamma$

$\mathcal{J}$ is linear multiplet $\Sigma$

$\rightarrow \int d^4\theta\, V_b \Sigma$

scalar in $V_b$ is FI term $\zeta$

$Z$ will get contribution $\sum q_i m_i$ with $m_J = \zeta$

$\rightarrow$ states with charge $q_J$ under $U(1)_J$ obey BPS bound $M \geq |q_J \zeta|$

<u>Chern-Simons couplings:</u>

$\sum_{i,j=1}^{r} \kappa_{ij} \int d^4\theta\, \Sigma_i V_j$ , where $\Sigma_i = \varepsilon^{\dot{\alpha}\beta} \bar{D}_{\dot{\alpha}} D_\beta V_i$

is supersymmetric completion of

$\sum_{i,j} \kappa_{ij} A_i \wedge dA_j$

Integrating charged fermions gives

$$(K_{ij})_{eff} = K_{ij} + \frac{1}{2} \sum_f (q_f)_i (q_f)_j \,\mathrm{sign}(M_f)$$

with $M_f = \tilde{m}_f + \sum_{i=1}^{r} (q_f)_i \, \phi_i$

Gauge invariance gives $(K_{ij})_{eff} \in \mathbb{Z}$

$\rightarrow$ quantization condition:

$$K_{ij} + \frac{1}{2} \sum_f (q_f)_i \, (q_f)_j \in \mathbb{Z}$$

<u>U(1) gauge theories</u>:

Consider U(1) gauge theory with $N_f$ flavors $Q^i, \tilde{Q}_{\tilde{i}}$ $(i, \tilde{i} = 1, \cdots, N_f)$, with charge $\pm 1$.

$\rightarrow$ 1d Coulomb branch: $\Phi = \phi + i\gamma$,

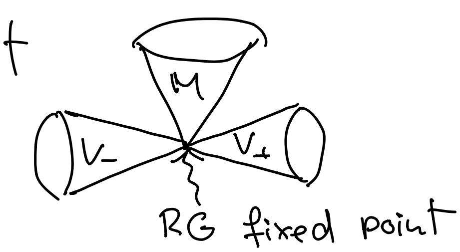
$\phi \in \mathbb{R}$, $\gamma \in S^1$ of period $2\pi g^2$

$\rightarrow$ for $N_f > 0$ there is $(2N_f - 1)$-dim Higgs branch parametrized by

$M^i_{\tilde{j}} = Q^i \tilde{Q}_{\tilde{j}}$ subject to $M^i_{\tilde{j}} M^k_{\tilde{\ell}} = M^k_{\tilde{j}} M^i_{\tilde{\ell}}$

Classically, the Higgs branch intersects the Coulomb branch at a point

$V_\pm = e^{\pm\Phi/g^2}$



$\phi > 0: \; V_+ \sim e^{\Phi/g^2}$

$\phi < 0: \; V_- \sim e^{-\Phi/g^2}$

global symmetries:

| | $U(1)_R$ | $U(1)_J$ | $U(1)_A$ | $SU(N_f)$ | $SU(N_f)$ |
|---|---|---|---|---|---|
| $Q$ | 0 | 0 | 1 | $N_f$ | 1 |
| $\tilde{Q}$ | 0 | 0 | 1 | 1 | $\overline{N_f}$ |
| $M$ | 0 | 0 | 2 | $N_f$ | $\overline{N_f}$ |
| $V_\pm$ | $N_f$ | $\pm 1$ | $-N_f$ | 1 | 1 |

Consider $N_f = 1 \longrightarrow$ at origin 3 branches meet

$\rightsquigarrow$ RG fixed point (SCFT)

There is a dual theory which flows to the same fixed point:

- 3 chiral fields $M, V_\pm$
- superpotential $W = -MV_+V_-$

$\rightsquigarrow$ leads to same moduli space!